– Circular Motion –

Centripetal Acceleration and Centripetal Force

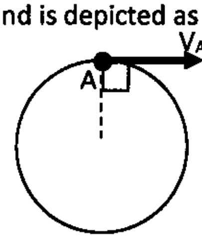
The Big Idea:

In this universe, if something is moving in a circle, there must be a force acting on it. Why? Because if something is moving and there is no force ($F_T = 0$), Newton's 1st Law tells us it must be moving at constant velocity, which means both constant speed and constant direction (Remember, velocity is speed and direction). Constant direction means a straight line. Circles aren't straight lines, so circular motion requires a force. It turns out, this force acts toward the center of the circle. More on that in a bit.

And again, in this universe, if there is a **force**, there must also be an **acceleration** in accordance with the 2nd Law, $F = ma$. This force is called centripetal force and this acceleration is also called centripetal acceleration. Since a force and its accompanying acceleration always act in the same direction, this acceleration must also point towards the center of the circle.

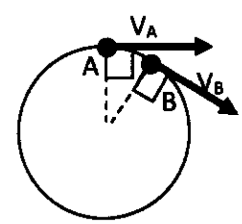
Concept 1: What is centripetal acceleration?

When something moves in a circle at a constant speed, it is actually accelerating toward the center of the circle even though its speed is not changing. This acceleration is called centripetal acceleration and is depicted as a_C. Centripetal means "center seeking". Here's the theory behind centripetal acceleration.



When an object moves in a circle, its velocity vector at any given instant is pointed along a line tangent to the circle as shown at right at point A for clockwise motion:

Recall from math class, a tangent touches the circle at the point in question, and is perpendicular (90°) to the radius (dashed line above).



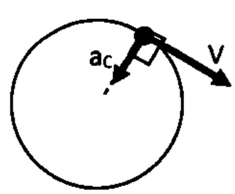
Now, consider the object moving along a circular path from A to B at a constant speed, as in the diagram at right. Its velocity changes from V_A to V_B because the direction changes, even though the speed remains constant.

Remember from back in September, that velocity is both speed and direction. Here, the direction changes, which is enough to change the velocity, even though the speed is constant.

If you do the vector parallelogram math for $\Delta V = V_B - V_A$, as at right, it turns out ΔV points along the radius toward the center of the circle.

So this means, when an object goes at constant speed from point A to point B along a circle, its **velocity change (ΔV)** is a vector which points radially inward, meaning along the radius, towards the center of the circle.

Again think back to September. **Acceleration is the change in velocity** in a given time: $a = \frac{\Delta v}{\Delta t}$. We just found ΔV. And Δt, being a scalar, doesn't change the direction of ΔV. This means **the acceleration vector points in the same direction as ΔV, towards the center of the circle (But NOT the same direction as V!!!).** This acceleration is called "centripetal acceleration" and is depicted as a_C.



The math is a little tedious, but the magnitude of the centripetal acceleration vector a_C is v^2/r, where v is the constant speed of the object and r is the radius of the circle.

Concept 2: What is Centripetal Force?

The second concept is centripetal force. As always, F = ma. Both F and a are vectors, but m is a scalar. In multiplication, scalars (here mass) don't do anything to change the direction of the vector they're multiplied with. That means F always points in the same direction as a. Like acceleration, centripetal force is depicted as F_C. So centripetal force F_C points towards the center of the circle just like a_C. Putting this all together yields:

As always, from Newton's 2nd Law:	$F = ma$	
In the case of circular motion:	$F_C = ma_C$	where both F_C and a_C point toward the center of the circle
Substituting $a_C = v^2/r$:	$F_C = mv^2/r$	and v points along the tangent of the circle (90° to the radius)

Where does this Centripetal Force come from?

If something is moving in a circle (at constant speed), there must be a force pointed towards the center of the circle. This force is centripetal force, or F_C. But it isn't some new type of force. One of the forces you already know about is acting as F_C. In other words, if something is moving in a circle, F_g, F_N, F_P, or F_f must be pointed towards the center of the circle. A big part of solving this class of problems is figuring out what is acting as F_C in a particular scenario.

Friction and turning cars

Back in the friction packet (Forces Packet 4) I said the force of friction acts:

parallel to the surfaces in contact
pointed so as to oppose motion or attempted motion.

I also said we would modify this slightly later. Later is now. In these scenarios involving objects going in a circle on a surface, the force of friction is still parallel to the surfaces in contact, but points toward the center of the circle. It is this force of friction which actually turns the car. It is static friction since the tire is not sliding over the road. (When the tire begins to slide, the car doesn't keep turning, but slides into the ditch)

Concept summary (source of many a quiz multiple choice question)

When an object moves along a circular path at a constant speed, its velocity is tangent to the circle. But its velocity is not constant even though its speed is constant. How can that be? Because <u>the direction</u> of velocity is changing.

The magnitude of velocity is: v (its speed)

The direction of velocity is: tangent to the circle

Since the velocity is not constant, it must be accelerating. This acceleration is called centripetal acceleration, a_C.

The magnitude of a_C is: $a_C = v^2/r$ where v is the speed and r is the radius of the circle

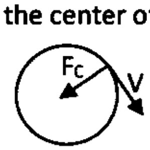
The direction of a_C is: radially inward (toward the center of the circle)

When an object moves along a circular path at a constant speed, the object must have a force acting on it (or else it would go in a straight line at a constant speed in accordance with Newton's 1st Law.) This force is called centripetal force, F_C.

The magnitude of F_C is: $F_C = ma_C = mv^2/r$

The direction of F_C is: radially inward. (toward the center of the circle)

So the velocity and centripetal force always point 90° from each other.



Problem solving techniques

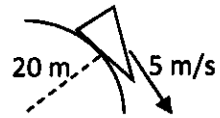
For the simplest problems in which you have 3 of the 4 variables, you can get away with just saying $F_C = mv^2/r$ and solving for the variable you're missing. Here's an example:

<u>Example 1</u>

An 800 kg car goes around a level corner at a constant speed of 5 m/s in a circular path. The radius of the circle is 20 m.

a) What is the acceleration of the car? Magnitude and direction.

Top View



$a_C = v^2/r$
$a_C = (5\ m/s)^2/20\ m$
$a_C = 1.25\ m/s^2$, radially inward

b) What is the centripetal force acting on the car?

$F_C = ma_C$
$F_C = (800\ kg)(1.25\ m/s^2)$
$F_C = 1{,}000\ kg \cdot m/s^2$ or 1,000 N, also radially inward

Note: You could get F_C in one step by using $F_C = mv^2/r = (800\ kg)\ (5\ m/s)^2/20\ m = 1{,}000$ N, radially inward.

Also note, this thought process does not give us any insight into what is causing the F_C. Remember, F_g, F_N, F_P, or F_f must be the "source" of F_C.

But most centripetal problems are more complicated than the previous simple example and require the general framework described on the next page.

Answers to selected questions

1.e.	$31.25\ m/s^2$	1.c	3.75 N
2.	0.31		
3.b.	2×10^{20} N		
4.	10 m/s		
5.	1.22 m/s		
6.e.	730 N	6.g	5.48 m/s
7.b.	9.46 m/s	7.c	9.46 m/s
8.	.66		
9.	9.10 m/s		
10.b.	1252.33 N		
11.	10.95 m/s for both		
12.	112.5 m		
13.	a. 16.73 m/s (about 37 mph) b. 12.65 m/s (about 28 mph) c. 8.94 m/s (about 20 mph) d. sqrt($a_g\mu r$)		

Step 1: Draw the Circular View.

This is the view in which the circular motion occurs in the plane of the paper. For a car driving in a circle, it is a "top view". For a roller coaster going around a loop, it is a "side view". etc. On this view, draw 3 things:

1. the circle or portion of the circle
2. the velocity vector (tangent to the circle, in the direction of motion)
3. the radius.

Step 2: Draw the FBD

This is the same FBD you've been drawing for months now. It is usually oriented to show gravity pointing straight down. On this FBD, show all forces existing in the scenario using Giraffe Necks are Pretty Flexible to identify F_g, F_N, F_T, and F_f. Remember, some of those may not be present, and there may be more than one of others - typically F_T.

F_C does not go on the FBD. F_C does not exist in and of itself. Some existing force already on your FBD must be pointing towards the center of the circle, thus acting as F_C.

Step 3: Identify which force on the FBD is acting towards the center of the circle and solve.

Look at your FBD and reconcile it with your Circular View. One (or more) of the forces on your FBD must be pointing towards the center of the circle, meaning along the radius in your Circular View. If there is not a force on your FBD pointing towards the center of the circle, your FBD is wrong. Figure it out and fix it.

Any of the force types we've talked about, F_g, F_N, F_T, and F_f, can be pointing towards the center of the circle. It's frequently a different force in different FBD's. You must think about the words of the scenario, compare your FBD to your circular view, and figure out which FBD force is pointing towards the center of the circle. This is the hard part of the problem.

Set this force(s) which point toward the center of the circle equal to F_C (and thus equal to mv^2/r) and solve.

Here's an example involving a car going around a level circular curve.

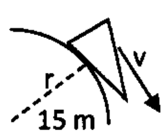
Example 2

A 700 kg car goes around a level corner at a constant speed in a circular path. The radius of the circle is 15 m. The coefficient of static friction is 0.7. How fast can the car go without sliding off the road?

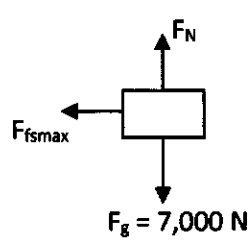
Step 1: Draw the Circular View

To put the circular motion on the paper, you must view the motion from the top. Identify v, r, and the circle

(Top View)



Step 2: Draw the FBD



(Front View)

Step 3: Identify which FBD force acts as F_C (points along radius) and solve

$F_{fsmax} = F_C$

$\mu_s \cdot F_N = m(v_{max})^2/r$

$(0.7)(7{,}000\text{ N}) = (700\text{ kg})((v_{max})^2/15\text{ m}$

$v_{max} = 10.25$ m/s

$\Sigma F_Y = 0$ (why 0?)

$F_N + F_g = 0$

$F_N - 7{,}000\text{ N} = 0$

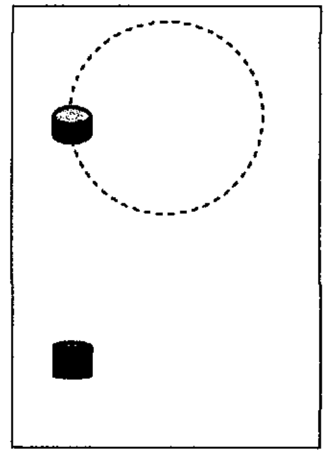
$F_N = 7{,}000$ N

F_f supplies the F_C here. (see pg 2) When you're driving around a corner as fast as you can, your tires are pushing on the road with the maximum available force of static friction (F_{fsmax}) and the road is pushing back with that same force (3rd Law). Thus $F_{fsmax} = F_C$.

Practice Questions (Selected answers back on page 3)

1. You have a 0.12 kg roll of tape attached to a 0.8 m long string. You whirl the string around your head in causing the roll of tape to travel in a horizontal circle at a constant speed of 5 m/s.

   a. Draw a "Circular View" of the motion. Depict the circle or arc, velocity vector and radius



   b. Draw an FBD "side view" of the roll of tape depicting all forces.

   c. What magnitude is the centripetal acceleration of the roll of tape?

   d. How strong is the Centripetal Force?

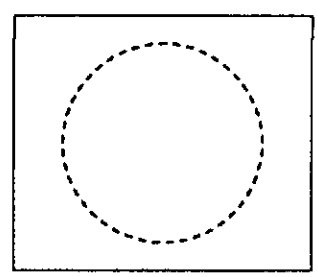
   e. What agent exerts the centripetal force on the tape?

   f. Describe the path that the roll of tape would take if you let go of the string.

2.

3. The 7.36×10^{22} kg moon travels in a circle (nearly) of radius 3.85×10^{8} m, at a constant speed of 1,023 m/s.

   a. Draw the circular view and FBD of the moon. What is providing the centripetal force on the moon? (what kind of force is it?)



   b. How strong is the centripetal force acting on the moon?

4. You drive your 700 kg car around a circular corner of radius 50 m at a constant speed. A centripetal force of 1,400 N acts on your car. How fast are you going?

5.

6. A popular amusement park ride is shown in Figure 2. It operates as follows: riders enter the cylindrical structure when it is stationary with the floor at the point marked "a". They then stand against the wall as the cylinder begins to rotate. When it is up to speed, the floor is lowered to the position marked "b", leaving the riders "suspended" against the wall high above the floor. The radius of the ride is 1.5 m, the mass of the rider is 73 kg and the coefficient of friction between the rider and the wall is 0.5.

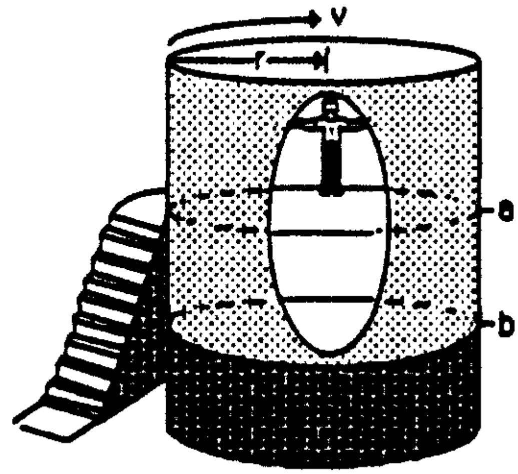


Figure 2

a. Consider the person as the object for the FBD

   i. In what direction does gravity act?

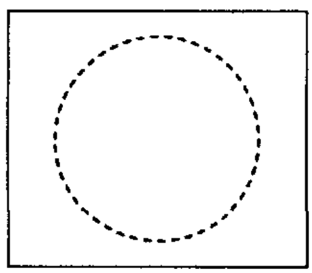
   ii. In what direction does friction act?

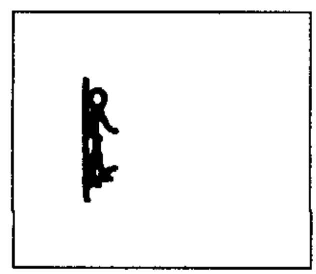
   iii. What surface provides the normal force?

   iv. In what direction does the normal force act?

   v. Draw the FBD for the person on the ride (at right)

b. Draw the circular view and FBD





c. From the FBD, write the equation for the total force in the y-direction (ΣF_Y)

d. From Newton's Laws, write the appropriate equation for ΣF_Y

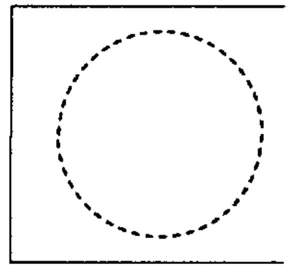
e. Using c. and d. solve for F_f

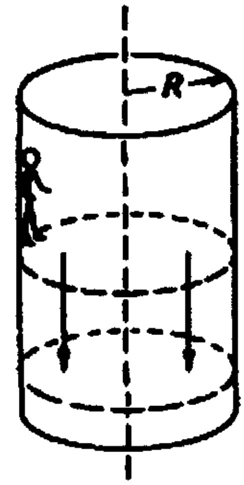
f. Which force (gravity, friction or normal force) provides the centripetal force?

g. Setting the force identified in (f.) equal to F_C (and thus equal to mv^2/r), what is the minimum speed required to keep the person from slipping?

7. A certain ride at an amusement park consists of a hollow cylinder that can rotate at high speeds. The floor can then be dropped with the people staying pinned to the sides of the cylinder. The cylinder has a radius of 6 meters and the coefficient of friction between the cylinder wall and a person is 0.67.

a. Draw the Circular View and FBD of the person





b. What is the minimum speed the cylinder must spin so that a 70 kg person will stay on pinned to the wall when the floor drops out?

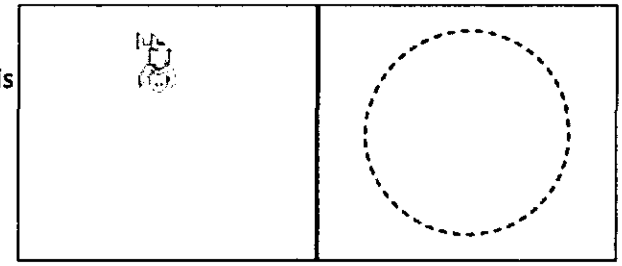
c. What is the minimum speed the cylinder must spin so that a 90 kg person will stay on pinned to the wall when the floor drops out? *You do not have to re-draw FBD and circular view*

8.

9.

10. You are watching a roller coaster at an amusement park which has a loop, meaning the track goes around a vertical circle such that the cars (and people!) are upside down at the top of the loop. The loop has a radius of 15 m. Ignore friction.

   a. Draw the Circular View and FBD of a 65 kg person at the top of the loop. (The person in a roller coaster car, but the FBD is only of the person)



   b. If the car (and thus also the person) is traveling at a speed of 17 m/s at the top of the loop, how strong is the centripetal force acting on the person?

   c. Which force or forces is (are) causing the centripetal force?

13. You are driving at a constant speed around a circular corner which has a 40 m radius.

   a. What is the fastest you can drive when the pavement is dry and the coefficient of friction between your tires and the road is 0.7?

   b. What is the fastest you can drive when the pavement is wet and the coefficient of friction between your tires and the road is 0.4? (you may start your work in part b. where a. above changes)

   c. What is the fastest you can drive when the pavement is snowy and the coefficient of friction between your tires and the road is 0.2? (you may start your work in part c. where a. above changes)

   d. What is the general relationship (variables, no numbers) between the maximum speed and the coefficient of friction in this scenario?